APPLICATION OF FUZZY LINEAR PROGRAMMING METHOD TO EXAMINE ROUTING OF PASSENGER TRAINS

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Abstract. The aim of this study is to develop a methodology for selection of routing of different categories of intercity passenger trains taking into account of the uncertainty of the processes and using the fuzzy linear programming method. The fuzzy theory allows for the presentation of real data with uncertainty due to imprecise measurement. This research studies the fuzzy problems with fuzzy constraints of passengers and with fuzzy triangular numbers as coefficients of the technological matrix that presents how full a train is. As a criterion of the optimization minimum direct operating costs are used. The restrictive conditions of the model include: satisfying the demand for railway transport by sections of the railway network; capacity of the sections; required minimum frequency; available compositions in exploitation; integer and positive decision. The fuzzy linear problem has been reduced to parametric linear programming. Taking into account the parameter, the limits of amendment of the optimal number of trains depending on passenger flows have been defined. The application of the methodology makes it possible to study the impact of the fluctuations in passenger flows on the number of trains. The decision approach proposed in this paper was tested for Bulgarian rail network and proposed an organization of railway passenger transport. The approach presented here can also be used for solving the scheme of transportation for other types of transport.

Keywords: fuzzy linear programming, passenger flows, routing, passenger trains capacity utilization coefficient.

Introduction

The development of an optimal transport plan of passenger trains on the railway network is related to a preliminary study of passenger flow, utilization of the train capacity and transport demand. Passenger flows are characterized by irregularity regarding hours, days of the week, months, and seasons. In many cases, passenger flows cannot be determined accurately, we cannot get enough real sample data to calculate the parameters by statistical ways. To deal with the problem in a mathematical way, these parameters can be treated as fuzzy variables. Fuzziness appears when the information is vague or is not clearly defined. The Fuzzy set theory allows for the description of real situations, taking into account the uncertainty of the processes. The fuzzy linear programming approach (FLP) is an extension of the linear programming (LP) approach that allows for the incorporation of the uncertainty factor in the construction of a mathematical model to increase the adequacy of the optimization. Fuzzy programming is more flexible and makes it possible to find solutions, which are more satisfactory for the real problem. In FLP, the uncertainty can be present in the objective function and/or in the constraints of the problem, and it can be represented by fuzzy sets. Many researchers have considered various types of the FLP problems and proposed several approaches for solving the FLP problems, [1-6].

In the field of railway transport the fuzzy linear optimization is used in [7-11]. A fuzzy optimization model is used for rescheduling high-speed railway timetable under unexpected interferences [7]. The model applies different parameters of the soft constraints with uncertainty margin to describe their importance for the purpose of optimization and treats the objective in the same manner. The model was experimented for Beijing-Shanghai high-speed rail line in China. In [8] is built the fuzzy linear programming model to solve the train pathing problem of the Beijing-Shanghai Railway. The authors design the fuzzy membership function to describe the triangular fuzzy coefficients. The objective function includes minimization of the transferring cost, running cost, and social adverse effect cost. The character of the coefficient of the costs is described with the fuzzy membership function. The constraints of the model include the segment and station capability. In [9] is proposed an optimization model that consists of three different objectives: the cost of train formation, the cost of idle time of waiting wagons in stations, and the cost of wagon classifications in shunting stations. The admissible maximum tonnage of trains in the planning stage is considered as a fuzzy number. In [10] in order to design a high-quality timetable in a fuzzy environment the train re-scheduling problem under the fuzzy environment is studied, in which the fuzzy coefficients of the constraint resources have the fuzzy boundaries. The railway section between Beijing and Zhengzhou in China is investigated in the research. In [11] is developed a multiobjective programming model for

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optimal allocation of passenger train services on an intercity high-speed rail line. The two objectives of the model are minimizing the operator's total operating cost and minimizing the passenger's total travel time loss. The model is solved by a fuzzy mathematical programming approach to determine the best-compromise train service plan, including the train stop-schedule plan, service frequency and the fleet size.

The fuzzy linear programming is applied also to make research in transport to solve the transportation problems [12]; in transport planning [13-15].

This paper aims to propose an approach to apply the fuzzy linear programming for selection of a transport plan for intercity trains by taking into account the fuzziness of passenger flows and passenger train capacity utilization. It would help take account of the uncertainty of the processes.

It can be summarized that in the reviewed studies related to a transportation plan the fuzziness in the FLP models was investigated for costs, tonnage of the trains, the time interval between a train leaving a station and another train arrival at the same station. The running time, dwell time, headway time for station and headway time for section were investigated by FLP with soft constraints. The fuzziness of passenger flows and passenger train capacity utilization are not investigated.

The novel contribution of this paper is the development of a methodology for selection of the optimal scheme for movement of intercity trains based on fuzzy integer linear programming (FILP), comparison of different variants of models of fuzzy problems, setting the passenger flows and the passenger train capacity utilization coefficient as fuzziness variables.

Materials and methods

Fuzziness can appear in different forms, i.e. with fuzzy inequalities, fuzzy objective function, both fuzzy inequalities and fuzzy objective function and fuzzy parameters.

In the research we have examined organization of intercity passenger trains according to the train categories and the number of wagons. The train categories are: Direct fast trains (DFT), Accelerate fast trains (AFT) and Fast trains (FT). Direct fast trains are intercity express trains. They are a new category of trains included in the research. Accelerate fast trains are intercity trains. Fast trains stop at more stations when compared to the DFT and AFT. The number of wagons is chosen according to the existing situation of formation of compositions on the Bulgarian rail network. The trains are composed of 4 wagons. A linear optimization model for these categories of trains have been used by the author to determine the number of trains in [16].

In this research the optimization of train services is made using the fuzzy set theory. The objective function presents minimization of the direct operational costs:

$$R = \sum_{i_1=1}^{I_1} r_{i_1}^{o,d,FT} . l_{i_1} x_{i_1}^{FT} + \sum_{i_2=I_1+1}^{I_2} r_{i_2}^{o,d,AFT} . l_{i_2} x_{i_2}^{AFT} + \sum_{i_3=I_2+1}^{I_3} r_{i_3}^{o,d,DFT} . l_{i_3} x_{i_3}^{DFT}, BGN/day \rightarrow \min \quad (1)$$

where $i_1 = 1,...,I_1$ – number of routes of fast trains;

$i_2 = I_1+1,...,I_2$ – number of routes of accelerate fast trains;

$i_3 = I_2+1,...,I_3$ – number of routes of direct fast trains;

$r_{i_1}^{o,d,FT}$, $r_{i_2}^{o,d,AFT}$, $r_{i_3}^{o,d,DFT}$ – direct operational costs for trains, BGN·km^{-1};

l_{i_1}, l_{i_2}, l_{i_3} – lengths of destination of trains, km;

x_{i_1}, x_{i_2}, x_{i_3} – numbers of trains of destinations respectively i_1, i_2, i_3.

The objective function (1) defines the optimal plan that provides the realization of the necessary passenger transportation with minimal direct operational costs.

The restrictive conditions are:

$$\sum\nolimits_{i_1=1}^{I_1} L_{i_1 jk}^{FT} \cdot \tilde{\alpha}_{i_1}^{FT} \cdot a_{i_1}^{FT} \cdot x_{i_1}^{FT} \gtrsim P_{jk}^{FT} \quad , \quad (2)$$

$$\sum\nolimits_{i_2=I_1+1}^{I_2} L_{i_2 jk}^{AFT} \cdot \tilde{\alpha}_{i_2}^{AFT} \cdot a_{i_2}^{AFT} \cdot x_{i_2}^{AFT} \gtrsim P_{jk}^{AFT} \quad , \quad (3)$$

$$\sum_{i_3=I_2+1}^{I_3} L_{i_3jk}^{DFT} \cdot \tilde{\alpha}_{i_3}^{DFT} \cdot a_{i_3}^{DFT} \cdot x_{i_3}^{DFT} \gtrsim P_{jk}^{DFT} \ , \qquad (4)$$

$$\sum_{i_1=1}^{I_1} L_{i_1jk}^{FT} \cdot x_{i_1}^{FT} + \sum_{i_2=I_1+1}^{I_2} L_{i_2jk}^{AFT} \cdot x_{i_2}^{AFT} + \sum_{i_3=I_2+1}^{I_3} L_{i_3jk}^{DFT} \cdot x_{i_3}^{DFT} < N_{jk}^{\max} \ , \qquad (5)$$

$$x_{i_1}^{FT} \geq M_{i_1} \, ; x_{i_2}^{AFT} \geq M_{i_2} \, ; \ x_{i_3}^{DFT} \geq M_{i_3} \, , \qquad (6)$$

$$\sum_{i_1=1}^{I_1} x_{i_1}^{FT} + \sum_{i_2=I_1+1}^{I_2} x_{i_1}^{FT} + \sum_{i_3=I_2+1}^{I_3} x_{i_1}^{FT} \leq W \ , \qquad (7)$$

$$x_{i_1}^{FT} \geq 0 \, ; x_{i_2}^{AFT} \geq 0 \, ; \ x_{i_3}^{DFT} \geq 0 \, ; x_{i_1}^{FT} \, ; x_{i_2}^{AFT} \, ; \ x_{i_3}^{DFT} \text{- integer} \qquad (8)$$

where $\tilde{\alpha}_{i_1}^{FT}, \tilde{\alpha}_{i_2}^{AFT}, \tilde{\alpha}_{i_3}^{DFT}$ – coefficients of utilization of seats (passenger train capacity utilization coefficient), are presented with triangular fuzzy numbers and have values in the interval (0, 1];

$a_{i_1}^{FT}, a_{i_2}^{AFT}, a_{i_3}^{DFT}$ – number of seats in a train;

$P_{jk}^{FT}, P_{jk}^{AFT}, P_{jk}^{DFT}$ – maximum passenger flow in a section formed between two adjacent stations j and k who will use fast trains, accelerate fast trains and direct fast trains respectively, passenger per day;

$L_{i_1jk}^{FT}$, $L_{i_2jk}^{AFT}$, $L_{i_3jk}^{DFT}$ – coefficients that take into account the possibility of the passenger train of route i_1, i_2, i_3 to serve the section formed between two adjacent stations j and k;

$L_{i_1jk}^{FT}=1$, $L_{i_2jk}^{AFT}=1$, $L_{i_3jk}^{DFT}=1$, where it is possible; $L_{i_1jk}^{FT}=0$, $L_{i_2jk}^{AFT}=0$, $L_{i_3jk}^{DFT}=0$, otherwise;

M_{i_1}, M_{i_2}, M_{i_3} – minimal number of trains for the routes of fast trains, accelerate fast trains and direct fast trains respectively, train/day;

$< N_{jk}^{\max}$ – maximum capacity of the railway line between two adjacent stations j and k, which is being examined, train/day;

W – available compositions in exploitation.

$\gtrsim$ – this symbol means fuzzy inequalities, i.e. that could permit some violations in the accomplishment of the constraints.

Conditions (2), (3) and (4) mean ensuring a seat for each passenger on any section of the railway network. Condition (5) means that the number of trains must not exceed the maximum capacity of the railway line. Condition (6) means that for some routes serving major transportation hubs, which are regional administrative centres, it is necessary to realize the satisfaction of certain frequency transport links. Condition (7) means realization of organization of train traffic with the available compositions in exploitation. Condition (8) means that the number of trains must be positive and integer.

The coefficients of utilization of seats present how full a train is. In the study it is defined as a fuzzy parameter and is given by triangular fuzzy numbers, which are represented with three points low (l), medium (m) and upper (u) for each of the investigated routes. The defuzzification is made by the average value of triangular fuzzy numbers.

In this research the optimization problem defined by formulas from (1) to (8) is in the field of fuzzy integer linear programming where part of a condition is with fuzzy inequalities and fuzzy numbers as coefficients of the technological matrix.

In the study the presented problem is solved by transforming FILP into a deterministic model with linear constraints. This problem is solved by linear programming and applying the method of parametric linear programming to have a solution.

To solve the problem the linear membership functions for fuzzy constraint are determined as follows:

$$\mu_{P_{jk}^{FT}} = \begin{cases} 1, \text{if } \sum_{i_1=1}^{I_1} L_{i_1 jk}^{FT} \cdot \tilde{\alpha}_{i_1}^{FT} \cdot a_{i_1}^{FT} \cdot x_{i_1}^{FT} > P_{jk}^{FT} \\ \dfrac{\sum_{i_1=1}^{I_1} L_{i_1 jk}^{FT} \cdot \tilde{\alpha}_{i_1}^{FT} \cdot a_{i_1}^{FT} \cdot x_{i_1}^{FT}}{d_{jk}^{FT}}, \text{if } P_{jk}^{q,FT} - d_{jk}^{FT} < \sum_{i_1=1}^{I_1} L_{i_1 jk}^{FT} \cdot \tilde{\alpha}_{i_1}^{FT} \cdot a_{i_1}^{FT} \cdot x_{i_1}^{FT} < P_{jk}^{FT} \\ 0, \text{if } \sum_{i_1=1}^{I_1} L_{i_1 jk}^{FT} \cdot \tilde{\alpha}_{i_1}^{FT} \cdot a_{i_1}^{FT} \cdot x_{i_1}^{FT} \le P_{jk}^{FT} - d_{jk}^{FT} \end{cases} \quad (9)$$

where $\mu_{P_{jk}^{FT}}$ – linear membership function for fuzzy constraint (2);

$d_{jk}^{FT}, d_{jk}^{AFT}, d_{jk}^{DFT}$ – values of the possible range of change of the restrictive conditions.

In a similar manner the linear membership functions $\mu_{P_{jk}^{AFT}}$, $\mu_{P_{jk}^{DFT}}$ for fuzzy constraint (3) and (4) are defined. Using the membership function defined in (9) and the similar linear membership functions for fuzzy constraints (3) and (4) the auxiliary parametric integer linear programming (APILP) problem is obtained. The objective function of APILP is the same as FILP given by formula (1). Conditions (2), (3) and (4) are transformed as follows:

$$\sum_{i_1=1}^{I_1} L_{i_1 jk}^{FT} \cdot \tilde{\alpha}_{i_1}^{FT} \cdot a_{i_1}^{FT} \cdot x_{i_1}^{FT} \gtrsim P_{jk}^{q,FT} - d_{jk}^{q,FT}(1-\beta) \quad (10)$$

$$\sum_{i_2=1}^{I_2} L_{i_2 jk}^{AFT} \cdot \tilde{\alpha}_{i_2}^{AFT} \cdot a_{i_2}^{AFT} \cdot x_{i_2}^{AFT} \gtrsim P_{jk}^{q,AFT} - d_{jk}^{q,AFT}(1-\beta) \quad (11)$$

$$\sum_{i_3=1}^{I_3} L_{i_3 jk}^{DFT} \cdot \tilde{\alpha}_{i_3}^{DFT} \cdot a_{i_3}^{DFT} \cdot x_{i_3}^{DFT} \gtrsim P_{jk}^{q,DFT} - d_{jk}^{q,DFT}(1-\beta) \quad (12)$$

$$9 < \beta \le 1 \quad (13)$$

where β – the parameter.

Conditions (5), (6), (7) and (8) remain unchanged.

Results and discussion

The proposed methodology has been applied for the railway network of Bulgaria. The twenty six intercity routes that serve the entire railway network have been investigated out of which 3 routes of DFT, 7 routes of AFT and 16 routes of FT. The number of wagons in the compositions of each category of trains is accepted to be equal to 4. This number corresponds to the real number of wagons.

Table 1 presents the initial and final station on the routes, the train category and the results of applying the methodology based on the auxiliary parametric integer linear programming.

Table 1

Table of routes of trains and results

Train routes			Triangular fuzzy numbers				Number of pair of train				Number of pair of train				
							l	m	u	$\tilde{\alpha}$	Parameter				
			l	m	u	$\tilde{\alpha}$	N_l	N_m	N_u	N_f	0.2	0.4	0.6	0.8	1
x1	Sofia - Plovdiv	DFT	0.55	0.65	0.8	0.67	1	1	1	1	1	1	1	1	1
x2	Sofia - Burgas	DFT	0.5	0.6	0.8	0.63	2	1	1	1	1	1	1	1	1
x3	Sofia - Varna	DFT	0.5	0.6	0.8	0.63	2	2	1	2	1	1	1	2	2
x4	Sofia - Plovdiv	AFT	0.55	0.65	0.8	0.67	1	1	1	1	1	1	1	1	1
x5	Sofia - Plovdiv - Burgas	AFT	0.5	0.6	0.8	0.63	1	1	1	1	1	1	1	1	1
x6	Sofia - Karlovo - Burgas	AFT	0.5	0.6	0.8	0.63	2	1	1	1	1	1	1	1	1

Table 1 (continued)

Train routes			**Triangular fuzzy numbers**				**Number of pair of train**				**Number of pair of train**				
							l	m	u	$\tilde{\alpha}$	**Parameter**				
			l	m	u	$\tilde{\alpha}$	N_l	N_m	N_u	N_f	0.2	0.4	0.6	0.8	1
x7	Sofia - Varna	AFT	0.5	0.6	0.8	0.63	2	2	1	2	1	1	2	2	2
x8	Sofia - Ruse	AFT	0.5	0.6	0.8	0.63	1	1	1	1	1	1	1	1	1
x9	Sofia - Vidin	AFT	0.5	0.6	0.8	0.63	2	2	1	2	1	1	1	1	2
x10	Sofia - Blagoevgrad	AFT	0.5	0.6	0.8	0.63	1	1	1	1	1	1	1	1	1
x11	Sofia - Plovdiv	FT	0.6	0.85	0.9	0.78	2	2	2	2	2	2	2	2	2
x12	Sofia - Plovdiv - Burgas	FT	0.6	0.8	0.9	0.77	3	2	1	2	2	2	2	2	2
x13	Sofia - Karlovo - Burgas	FT	0.6	0.8	0.9	0.77	2	2	3	2	2	2	2	2	2
x14	Sofia - Plovdiv - Varna	FT	0.6	0.8	0.9	0.77	1	1	1	1	1	1	1	1	1
x15	Sofia - Karlovo - Varna	FT	0.6	0.75	0.9	0.75	1	1	1	1	1	1	1	1	1
x16	Plovdiv - Gorna Oryahovitsa - Varna	FT	0.6	0.8	0.9	0.77	1	1	1	1	1	1	1	1	1
x17	Sofia - Varna	FT	0.6	0.8	0.9	0.77	3	2	2	2	2	2	2	2	2
x18	Ruse - Varna	FT	0.6	0.8	0.9	0.77	3	2	2	2	2	2	2	2	2
x19	Sofia - Gorna Oryahovitsa	FT	0.6	0.85	0.9	0.78	1	1	1	1	1	1	1	1	1
x20	Sofia - Ruse	FT	0.6	0.8	0.9	0.77	1	1	1	1	1	1	1	1	1
x21	Plovdiv - Ruse	FT	0.6	0.8	0.9	0.77	2	1	1	1	1	1	1	1	1
x22	Sofia - Vidin	FT	0.6	0.8	0.9	0.77	4	3	3	3	3	3	3	3	3
x23	Sofia - Lom	FT	0.6	0.85	0.9	0.78	1	1	1	1	1	1	1	1	1
x24	Sofia - Kyustendil	FT	0.55	0.65	0.8	0.67	3	2	2	2	2	2	2	2	2
x25	Sofia - Kulata	FT	0.6	0.85	0.9	0.78	4	3	3	3	3	3	3	3	3
x26	Plowdiv - Varna	FT	0.6	0.75	0.9	0.75	1	1	1	1	1	1	1	1	1
Total number of pair of trains							48	39	36	39	36	36	37	38	39

For the routes that pass on parallel rail lines the intermediate station is shown. The number of sections, for which conditions (10), (11) and (12) are formed, is 31. The triangular fuzzy numbers for the passenger train capacity utilization coefficient of each assignment are determined taking into account the passenger train capacity utilization, which allows for covering the operating costs from the revenue. If the number of trains is determined at low passenger train capacity utilization coefficient, the number of trains will large and the operational costs will be high. If the number of trains is determined at high passenger train capacity utilization coefficient, the number of trains will be small and the transport needs of passengers will not be met. In this research the problem has been solved by setting the triangular fuzzy numbers of the passenger train capacity utilization coefficient.

In Table 1 the triangular fuzzy numbers for each route can be seen. In the columns of the table designated as N_l, N_m, N_u and N_f the results can be seen for the number of the trains at the low, medium, upper and defuzzification values of the passenger train capacity utilization coefficient. In these cases the fuzzy inequalities are not taken into account. In the last columns of Table 1 the values of trains according the value of the parameter are presented. These cases take into account the fuzziness of the capacity utilization coefficients. The values of the parameter β have been changed from 0.2 to 1 with step 0.2.

Fig. 1 presents the results for the operating costs and the number of trains taking into account the variants of the parameter *β*.

The results show:

- The number of trains per day and the operating costs per day remain unchanged for the values of the parameter *β* to 0.4.
- When changing the values of the parameter *β* from 0.6 to 1 with step 0.2 the number of trains and the operating costs increase.
- The limit value *β* = 1 corresponds to the solution when the fuzziness of passenger flows is not taken into account.

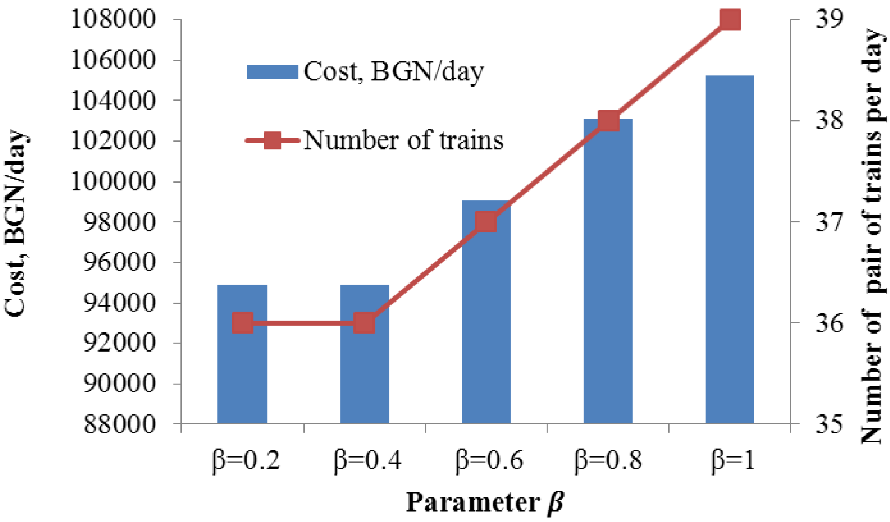

Fig. 1. **Reconsideration of decision, as value of parameter *β***

Conclusions

1. In the present study a methodology for selection of the optimal scheme for intercity train services has been development based on fuzzy integer linear programming. After the defuzzification the problem is solved by auxiliary parametric integer linear programming.
2. The passenger train capacity utilization coefficient is presented with triangular fuzzy numbers, the passenger flows are presented by fuzzy inequalities and the possible range of change of the values. This corresponds to a real situation, which is characterized by variability in the data.
3. The application of the methodology makes it possible to study the impact of the fluctuations in passenger flows on the number of trains. The limits of the parameters of amendment that produce the different solutions have been defined.
4. The impact of the capacity utilization of the train on the number of trains is also studied.
5. The method can be used for predictive solutions depending on the change of passenger flows.
6. The obtained values for the number of trains depending on the possible change of passenger flows can be used for finding operational technological solutions and also short-term forecasts in railway passenger transport.
7. Further study concerning the proposed methodology will be made to deal with the fuzziness of the routes of trains.

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